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1 Could the dynamics of field equations represent the dynamics for solving them?

The scaling hierarchies defined by powers of Φ, e, and primes p (possibly other hierarchies too) probably reflect something very profound. The observation that the discrete dynamics of Fibonacci numbers is dual with the continuous dynamics of simple second order differential equation suggests that some kind of duality might be in question and hints also a connection with what I call self referentiality.

One manner to formulate self referentiality is to say that the laws of physics must be such that the solutions of field equations in short length scales represent the process of solving field equations in long scales and vice versa. Field equations would thus represent the dynamics of solving field equations. This would guarantee that Universe can understand itself. For instance, the short scale dynamics of brain would automatically solve the dynamics in much longer length scales and it would be enough to become conscious about this solution.

This is of course just what we are doing when we construct theories and solve the field equations. This could explain also the miraculous number theoretical feats of some mathematicians and the ability of persons known as idiot savants to perform incredibly complex calculations, such as decomposing large integers to primes without having no idea what the notion of prime even means.

The idea of self referentiality inspires more concrete questions.

a) p-Adic length scale hypothesis states that preferred p-adic primes p satisfy $p \simeq 2^k$, k prime. The interpretation is that two length scales are involved: L_p and much shorter length scale L_k. Could the dynamics in the short p-adic length scale L_k automatically represent the procedure of solving field equations exponentially longer length scale L_p? Is the p-adic length scale hypothesis forced by the requirement that the dual dynamics of field equations and of the procedure of solving them are realizable physically as dual aspects of one and same dynamics?

b) Discrete Fibonacci dynamics represents as a short length scale dynamics the procedure of solving the Fibonacci differential equation. Is the importance of Golden Mean forced by the fact that these dual dynamics are realizable physically?

1.1 The two dynamics associated with Fibonacci numbers

1.1.1 Fibonacci series from a model of population dynamics

Fibonacci series arise from the model for the evolution of population assuming that time is discrete and time units is 1/2 of the average time for replication and members have infinitely long life time. This gives at time $t = n$

$$N(n) = N(n-1) + N(n-2) \ .$$

This is the general Fibonacci equation. The solution of the equation can be written as

$$N(n) = a \times \Phi^n + b \times (-1)^n \Phi^{-n} \ .$$

The two terms increase/decrease exponentially and the solution can be written as

$$N(n) = a \times e^{n/t_+} - b \times (-1)^n \times e^{-n/t_+} \simeq a \times e^{n/t_+} \ , \quad t_+ = \frac{1}{log(\Phi)} \ .$$

The approximation is excellent for large values of time $t = n$ since second term goes to zero exponentially. a and b are constants depending on initial values $N(1)$ and $N(0)$. As a special case one obtains various Fibonacci series when $N(1)$ and $N(0)$ are integers.

As expected, there is an exponential growth with time constant $\tau_+ = 1/log(\Phi)$, when time unit corresponds to half of the replication time. This model could explain why Fibonacci series is associated with biological growth. The model does not explain the the appearance of Golden Mean in the purely geometric context. For instance, why angle $2\pi/10$ for which cosine and sine are expressible in terms of Φ appears in DNA strand as the angle of twist associated with single DNA triplet along double strand axis.

1.1.2 Fibonacci numbers from differential equation

A basic fact about calculus is that second order differential equations, in particular the equation

$$\frac{d^2f}{dt^2} - \frac{df}{dt} - f = 0$$

can be solved using Taylor series expansion

$$f(t) = \sum_n f_n \frac{t^n}{n!} \ .$$

In this particular case one obtains a recursion formula

$$f(n) = f(n-1) + f(n) \ ,$$

which is Fibonacci equation again so that one can assign to the population dynamics a dual dynamics with respect to continuous time.

The solution is

$$f(t) = a \times e^{\Phi t} + b \times e^{t/\Phi} \ , \quad \Phi = \tfrac{1+\sqrt{5}}{2} \ .$$

What is intriguing that one obtains an exponential dynamics also now but with time constants

$$\begin{array}{l} \tau_+ = \frac{1}{\Phi} \ \text{(exponential amplification)} \ , \\ \tau_- = \Phi \ \text{(exponential amplification but with a slower rate)} \ . \end{array}$$

In the previous case the time constants are logarithms of these time constants

$$t_+ = \frac{1}{log(\Phi)} \ , \ t_- = -t_+ = log(\tau_-) \ .$$

Both dynamics are exponential but time scales are logarithmically related.

1.2 p-Adic scaling laws and generalized Golden Mean

The next observation is that p-adic scaling laws reduce to what might be called a generalized Golden Mean using a simple generalization of Fibonacci differential equation. Also the notion of Golden Mean map emerges and one can say that Golden Mean of Golden Mean is Golden Mean. Stated otherwise, Golden Mean is a fixed point of Golden Mean map. Fractals correspond to fixed points of maps and now the map would be Golden mean map. Large p-adic primes p variants are approximate fixed points of Golden Mean map.

1.2.1 Slight generalization of Fibonacci differential equation

The generalization of Fibonacci differential equation is given by

$$\frac{d^2f}{dt^2} - \gamma\frac{df}{dt} - f = 0 \ .$$

Now $f(t)$ is given by

$$f(t) = a \times e^{t/\tau_+} + b \times e^{t/\tau_-} \ \ , \quad \tau_\pm = \frac{\gamma \pm \sqrt{\gamma^2+4}}{2} \ \ , \quad \tau_+ = \frac{1}{\tau_-} \ \ .$$

τ_+ can be regarded as a generalization of Golden Mean and one can write $\tau_+ = \Phi(\gamma)$.

The coefficients of the Taylor expansion

$f(n)$ satisfy the equation

$$f(n) = \gamma \times f(n-1) + f(n-2)$$

generalizing the population dynamics. The general solution is

$$\begin{aligned} f(n) &= a \times \tau_+^n + b \times \tau_+^{-n} \\ &= a \times e^{n/t_+} - b \times (-1)^n \times e^{-n/t_+} \ \ , \quad t_+ = \frac{1}{log(\tau_+)} \ \ . \end{aligned}$$

1.2.2 p-Adic scaling laws and generalized Golden Means associated with primes

Consider as a special case the p-adic scaling laws with $\tau_+ = p$. The choice $\gamma = (p^2-1)/p$ gives

$$f(n) = a \times p^n + b \times p^{-n} \ \ ,$$

which corresponds to the p-adic scaling law.

The generalized Fibonacci numbers associated with primes p are given by

$$\Phi(p) = \frac{b \pm \sqrt{b^2+4}}{2} \ , \quad b = \frac{p^2-1}{p} \ .$$

and determines the time constants of the continuum dynamics. For large primes p one has $\Phi(p) \simeq p$ meaning asymptotically exponential behavior exp(pt).

1.2.3 Golden Mean of Golden Mean is Golden Mean

One can also consider the replacement of p with an arbitrary argument x and call the correspondence $x \rightarrow \Phi(x)$ Golden Mean map.

What is amazing that if one looks for the value of x giving $\Phi(x) = \Phi$ one finds $x = \Phi$. Golden Mean of Golden Mean is Golden Mean! Golden Mean would be fixed point of map $x \rightarrow \Phi(x)$. Rather generally, fractals correspond to fixed points of discrete dynamical maps and it would seem that Golden Mean map is especially important map dynamical map of this kind.

For large values of x, arbitrary x is an approximate fixed point of Golden Mean map by $\Phi(x) \simeq x$ for $x \rightarrow \infty$. Hence large p-adic primes would be also approximate fixed points of Golden Mean map (electron corresponds to $M_{127} = 2^{127} - 1 \simeq 10^{38}$).

1.3 What is the interpretation of the duality between discrete and continuum dynamics?

Differential equations define a pair of dual dynamics: continuous dynamics in continuous time domain t and discrete dynamics with respect to the fictive discrete time axis with time values $t = n$ labelling the Taylor coefficients $f(n)$ of $f(t)$. This generalizes also to field equations: now only the time parameter t is replaced with four space-time coordinates and Taylor coefficients are labelled by four integers.

1.3.1 Questions

The questions are following.

a) The discrete Fibonacci dynamics represents the dynamics of solving the Fibonacci differential equation with time scale which is exponentially longer. Could it be that also in Nature discrete Fibonacci dynamics represents the process of solving dynamics in longer scales?

b) Could this apply also in the case of p-adic fractality with powers of preferred primes $p \simeq 2^k$, k prime, replacing powers of Φ? In TGD framework the explanation of the p-adic scaling law has indeed been that there are two dynamics involved. The long scale p-adic dynamics with characteristic p-adic length scale L_p and the short scale p-adic dynamics corresponding to the dynamics with a characteristic p-adic length scale L_k, k prime. L_k is essentially the 2-based logarithm of L_p.

The question is whether the dynamics associated with the p-adic length scale L_p be analogous to the dynamics with respect to continuous time t and whether the dynamics associated with L_k could correspond to the discrete dynamics in the space of Taylor coefficients of this differential equation.

c) A more general question is whether the proposed correspondence could be true for field equations quite generally.

i) Mathematically this would mean that the dynamical variables effectively transforms from $f(t)$ to Taylor coefficients f_n as one moves from long length scales to short length scales. This transition is somewhat analogous to the transition from a function to its Fourier transform.

ii) Physically this would mean that the discrete short scale dynamics, say that of our brain, would represent the dynamics for the process of solving the field equations. This would be very nice since would guarantee that the Universe can understand itself and models itself automatically. The idea that Universe is performing mimicry at all possible levels is indeed one of the basic ideas of TGD inspired theory of consciousness, and could be called self referentiality.

iii) Self referentiality would imply that field equations represent the dynamics for the process of solving them. Long length scale dynamics would represent short length scale dynamics and vice versa.

1.3.2 p-Adic length scale hypothesis and self-referentiality

The hypothesis that the field equations represent also the procedure solving the field equations in longer length scales could allow to interpret p-adic length scale hypothesis.

In this case the the discrete time variable $t = n$ could correspond to a discrete valued time variable varying in the range $[0, k^n]$. The continuous variable t would correspond to dynamics in much longer time with the values of time variable in the range $[0, p^n]$. This is certainly possible since in principle Taylor series at single point contain all the relevant information.

Golden Mean p-adic fractalities would be associated with short length scale representations of the long length scale dynamics. Bio-systems indeed represent in their short length scale dynamics (cell, brain) the long length scale dynamics of the external world.

1.3.3 How this picture relates to the notion of rational physics?

The number theoretic vision about physics relies on the idea that physics is basically rational. This means that space-time surface, induced spinors at space-time surface, configuration space spinor fields, S-matrix, etc..., can be obtained by algebraically continuing their values in a discrete subset of rational variant of the geometric structure considered to appropriate completion of rationals (real or p-adic).

Only subset of rational points is acceptable since the simultaneous requirements of real and p-adic continuity can be satisfied only if one introduces ultraviolet cutoff length scale. This means that the distances between subset of rational points fixing the dynamics of the quantities involved are above some cutoff length scale, which is expected to depend on p-adic number field R_p. The continued quantities coincide only in this subset of rationals but not in shorter length scales.

The presence of the cutoff implies that the dynamics at short scales becomes effectively discrete. The natural identification of this effectively discrete dynamics would be as the counterpart of the discrete dynamics for the Taylor coefficients for the physically acceptable solutions of field equations or something analogous to it, say the dynamics of discrete Fourier coefficients. One might hope that partial difference equations would replace partial differential equations in the case of space-time surfaces at this limit so that the discrete subset of rational points would correspond to integer valued labels of Taylor coefficients, Fourier components, or something more general. Short distance physics would represent in very metaphorical sense the Fourier transform of long distance physics and vice versa.