INCORPORATION OF THE GOLDEN RATIO PHI INTO THE SCHRÖDINGER WAVE FUNCTION USING THE PHI RECURSIVE HETERODYNING SET

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Abstract. The Golden Ratio Phi (Φ) is an extraordinary and ubiquitous irrational number having a value of 1.618033… Phi's presence may be seen in both the biological and astronomical realms and recently in the quantum mechanical and physical realms. In the biological realm, the number Phi can be seen in both Phyllotaxis and DNA. In the astronomical realm, its presence is found in the spiral structures of galaxies. In physics, Phi can now be related to the g-factors of the electron, proton and neutron. This paper will show and prove that Phi is also intimately related to the quantum realm by virtue of its presence in the quantum mechanical wave function Ψ(x, y, z, t). The basis for the compact incorporation of Φ into the wavefunction will be derived from solving the Schrödinger Wave Equation and the use of the Phi recursive heterodyning set of wavelengths λ_n. Solutions to the Schrödinger Wave Equation based on these recursive wavelengths and Φ will be derived in both Cartesian and Polar coordinates. The state function derived from solving the Schrodinger Wave Equation is a compact relationship that includes the "four basic constants" (b4C) **2**, ***e***, ***π*** and **Φ** originally proposed by Michael Heleus [1] whereby he had shown an interesting relationship between the b4C in relation to the building of both the great pyramid of Giza, Egypt and the Parthenon in Athens, Greece. Heleus has postulated that these ancient builders erected structures based on two orthogonal axes. The numbers Phi and two for the North-South axis and the numbers ***e*** and ***π*** for the East-West axis. Heleus also found that the b4C are coordinated by a rule of exponents such that a new constant is created which is the least-mean-square error optimized value of the number which is simultaneously a root of each of the 4 constants whereby the index of the root is very close to an integer of 3 digits or less. This optimized value Heleus designated as HC (Heleus' constant) equal to 1.0060427, which is simultaneously approximately the 80th root of phi, the 115th root of 2, the 166th root of e, and the 190th root of pi. If all of these root indices are added up and then divided by four, the number 137.75 is obtained. The fine-structure constant α is equal to the reciprocal of 137.03599911. The difference between the fine-structure constant and the reciprocal of 137.75 only amounts to about 0.004% and is therefore well within the bounds of being scientifically significant.

Keywords: Schrödinger wave equation, Phi, heterodyning set, quantum mechanics, state function, b4C

1. Introduction:

The Golden Ratio Phi (Φ) has been termed "The World's Most Astonishing Number" by author Mario Livio in his book **<u>THE GOLDEN RATIO</u>**, Broadway Books, New York, 2002 [2]. Phi can easily be derived by solving the simple quadratic equation, $x^2 - x - 1 = 0$. The two roots of this equation are Φ = 1.6180333… and ϕ = -0.618033… or $x = \frac{1 \pm \sqrt{5}}{2}$. There are many hundreds, if not thousands of relationships involving Phi. The irrational number Φ arises from the famous Fibonacci Series whereby each successive number of the series is equal to the sum of the two preceding it (i.e., 0, 1, 1, 2, 3, 5, 8, 13, 21, 34…). Zero represents the 0th Fibonacci number F(0), one represents the first Fibonacci number F(1) and 34 represents the ninth Fibonacci number F(9). It is interesting to note that the first and second

Fibonacci numbers are both equal to one. The series rather quickly converges towards the numerical value of Φ when a number of the series is divided by the previous number of the series (i.e., 34/21 = 1.61905). Obviously, the farther out the series is taken, the closer will be the value of Φ when the quotient F(n)/F(n-1) is calculated. Some other interesting exact relationships to Phi are the following: $(\Phi + 1) = \Phi^2$; $Sin\left(i\ln\Phi\right) = i/2$; $Sin\left(\frac{\pi}{2} - i\ln\Phi\right) = \frac{\sqrt{5}}{2}$; $\Phi = 2Cos\left(\pi/5\right)$ and $Sin^2(i\ln\Phi) = -\frac{1}{4}$.
Additionally, no other number besides Phi is known to have the property that when it is added to one it exactly equals the square of itself.

2. The Heterodyning Set of Wavelengths

The heterodyning set of wavelengths may be expressed in the following form [3]:

(1) $$\lambda_n = \lambda_{n+1} + \lambda_{n+2} + \ldots\lambda_\infty$$

Expression (1) simply defines the heterodyning set as an infinite summation of wavelengths. It states that a single wavelength is actually the sum of an infinite number of wavelengths. Now, if we introduce the mathematical concept of scale invariance we may create any particular scale factor we wish according to the following [4]:

(2) $$\text{f (scale factor)} = \frac{\lambda_n}{\lambda_{n+1}} = \frac{\lambda_{n+1}}{\lambda_{n+2}} = \frac{\lambda_{n+2}}{\lambda_{n+3}}\ldots$$

The scale factor f is arbitrary and could be set to any number whatsoever. Now, we can allow each individual wavelength to be equal to an infinite sum of wavelengths as shown in equation (3) below:

(3) $$\text{Let } \lambda_n = \sum_{i=1}^{\infty}\lambda_{i+1} \text{ and } \lambda_{n+1} = \sum_{i=1}^{\infty}\lambda_{i+2} \text{ and } \lambda_{n+2} = \sum_{i=1}^{\infty}\lambda_{i+3} \text{ for all n} \geq 1$$

Likewise, the following relationship of the scale factor f therefore follows:

(4) $$\text{f} = \frac{\lambda_{n+1}}{\lambda_{n+2}} = \frac{\sum_{i=1}^{\infty}\lambda_{i+2}}{\sum_{i=1}^{\infty}\lambda_{i+3}} \text{ for n} \geq 1$$

Additionally, the following relationship logically can be obtained using equation (1):

(5) $$\lambda_n = \sum_{i=1}^{\infty}\lambda_{i+2} + \sum_{i=1}^{\infty}\lambda_{i+3} = \lambda_{n+1} + \lambda_{n+2}$$

We now may set up an equality ratio, cross-multiply and solve for λ_n [5]:

(6) $$\frac{\lambda_n}{\lambda_{n+1}} = \frac{\lambda_{n+1}}{\lambda_{n+2}} \text{ and therefore } \lambda_n = \frac{\lambda_{n+1}^2}{\lambda_{n+2}}$$

Now, if we substitute the above equation (6) into equation (1) we obtain the following quadratic equation [6]:

(7) $$\frac{1}{\lambda_{n+2}}\left(\lambda_{n+1}^2\right) - \lambda_{n+1} - \lambda_{n+2} = 0. \text{ Using the quadratic formula } \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

to solve for λ_{n+1} we obtain the following roots: $\lambda_{n+1} = \dfrac{\frac{1 \pm \sqrt{1+4}}{2}}{\lambda_{n+2}}$ or $\lambda_{n+1} = \Phi\lambda_{n+2}$ and $\lambda_{n+1} = -\phi\lambda_{n+2}$

where Φ = 1.618033988… and ϕ = 0.618033988…

The infinitely recursive wavelengths may now be expressed in terms of Φ in the following manner:

(8) $$\sum_{i=1}^{\infty} \lambda_{i+2} = \Phi \sum_{i=1}^{\infty} \lambda_{i+3} \text{ or } \lambda_{n+1} = \Phi\lambda_{n+2}$$

Also, substitution into equation (1) gives:

(9) $$\sum_{i=1}^{\infty} \lambda_{i+1} - \Phi \sum_{i=1}^{\infty} \lambda_{i+2} - \sum_{i=1}^{\infty} \lambda_{i+3} = 0 \text{ or } \lambda_n - \lambda_{n+1} - \lambda_{n+2} = 0;\ \lambda_n - \Phi\lambda_{n+2} - \lambda_{n+2} = 0$$

$$\lambda_n - (\Phi + 1)\lambda_{n+2} = 0 \text{ and since } (\Phi + 1) = \Phi^2 \text{, we therefore obtain } \lambda_n - \Phi^2\lambda_{n+2} = 0.$$

In summation form we obtain the following:

(10) $$\sum_{i=1}^{\infty} \lambda_{i+1} - \Phi^2 \sum_{i=1}^{\infty} \lambda_{i+2} = 0$$

In integral form, the above expression (10) becomes:

(11) $$\sum_i \left(\int_1^{\infty} \lambda_{i+1} d\lambda - \Phi^2 \int_1^{\infty} \lambda_{i+2} d\lambda \right) = 0$$

In double integral form the above expression (11) reduces to the double integral:

(12) $$\int_1^{\infty} \int_{\lambda_1}^{\lambda_\infty} \left(\lambda_{i+1} - \Phi^2 \lambda_{i+2} \right) d\lambda\, di = 0$$

3. The Time-Dependent Schrödinger Wave Equation [6]:

The ***Time-dependent Schrödinger wave equation*** for a one particle system in one dimension is:

(13) $$-\frac{\hbar}{i}\frac{\partial\Psi\left(x,t\right)}{\partial t}+\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\Psi\left(x,t\right)-V\left(x,t\right)\Psi\left(x,t\right)=0$$

where $\Psi\left(x,t\right)$ is the time-dependent state function of a particle moving in a one dimensional Cartesian Coordinate system and $V\left(x,t\right)$ is the potential energy function of the particle. Now, restricting ourselves to the special case where the potential energy function $V\left(x,t\right)$ is only a function of x, we obtain the following equation:

(14) $$-\frac{\hbar}{i}\frac{\partial\Psi\left(x,t\right)}{\partial t}+\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\Psi\left(x,t\right)-V\left(x\right)\Psi\left(x,t\right)=0$$

Using the separation of variables technique and letting $\Psi\left(x,t\right)=f\left(t\right)\psi\left(x\right)$ where $\psi\left(x\right)$ is the time-independent state function that is only dependent on x, we can solve the above expression (14) as follows:

(15) $$\frac{\partial\Psi\left(x,t\right)}{\partial t}=\frac{df\left(t\right)}{dt}\psi\left(x\right) \text{ and } \frac{\partial^2\Psi\left(x,t\right)}{\partial x^2}=f\left(t\right)\frac{d^2\psi\left(x\right)}{dx^2}$$

Substitution of (15) into the original equation (14) gives:

(16) $$\frac{-\hbar}{i}\frac{df\left(t\right)}{dt}\psi\left(x\right)+\frac{\hbar^2}{2m}f\left(t\right)\frac{d^2\psi\left(x\right)}{dx^2}-V\left(x\right)f\left(t\right)\psi\left(x\right)=0$$

Dividing equation (16) by $f\left(t\right)\psi\left(x\right)$ we obtain the following simplified differential equation:

(17) $$\frac{-\hbar}{i}\frac{1}{f\left(t\right)}\frac{df\left(t\right)}{dt}=-\frac{\hbar^2}{2m}\frac{1}{\psi\left(x\right)}\frac{d^2\psi\left(x\right)}{dx^2}+V\left(x\right)$$

Now, if we equate the left-hand side of equation (17) to the constant E (total energy of the system), we obtain the following equation:

(18) $$\frac{df(t)}{f(t)} = \frac{iE}{\hbar}dt$$

Upon integration of both sides of equation (18) we obtain the following relationship:

(19) $\ln f(t) = \frac{iEt}{\hbar} + C$ where C is an arbitrary constant of integration. Converting this equation in exponential form we obtain:

(20) $f(t) = e^c e^{\frac{-iEt}{\hbar}} = Ae^{-iEt/\hbar}$ Since A is an arbitrary constant of integration, it can thus be included as a factor in the function $\psi(x)$ and can be omitted from $f(t)$. Thus equation (20) may be simplified to:

(21) $$f(t) = e^{-iEt/\hbar}$$

4. Time-Independent Schrödinger Wave Equation [7]:

Now, if we equate the right side of equation (17) to E we obtain the ***Time-independent Schrödinger wave equation:***

(22) $$\frac{-\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + V(x)\psi(x) = E\psi(x)$$

The above equation (22) describes the motion of a single subatomic particle of mass ***m*** moving in one dimension ***x***. Thus, for cases where the potential energy is a function of ***x*** only, there exist wave functions of the form:

(23) $\Psi(x,t) = e^{-iEt/\hbar}\psi(x)$. Since $E = \frac{hc}{\lambda}$ and $\hbar = \frac{h}{2\pi}$ equation (23) can be re-written as

(24) $\Psi(x,t) = e^{-2\pi ict/\lambda}\psi(x)$. Normalization of the wave-functions, of course, requires that multiplication of each wave-function by its complex conjugate (Ψ^* and ψ^*) satisfy the following relationship: $\left|\Psi(x,t)\right|^2 = \left|\psi(x)\right|^2$. We may now therefore create a series of wave-functions based on the recursive wavelengths:

(25) $\Psi_i(x,t) = e^{-2\pi ict/\lambda_i}\psi_i(x)$ where naturally then $\ln \Psi_i(x,t) = \frac{-2\pi ict}{\lambda_i} + \ln \psi_i(x)$.

Or, however, in terms of n-recursive wave-functions where n =1, 2, 3,…∞.

(26) $\Psi_n(x,t) = e^{-2\pi ict/\lambda_n}\psi_n(x)$ and $\ln\Psi_n(x,t) = \dfrac{-2\pi ict}{\lambda_n} + \ln\psi_n(x)$.

Multiplying equation (26) by λ_n we obtain $\lambda_n\left(\ln\Psi_n(x,t) - \ln\psi_n(x)\right) + 2\pi ict = 0$ or

(27) $\lambda_n\left[\ln\dfrac{\Psi_n(x,t)}{\psi_n(x)}\right] + 2\pi ict = 0$. From the previously derived recursive wavelength equations

we obtain the following relationship: $\left[(\lambda_{n+1}) + (\lambda_{n+2})\right]\left[\ln\dfrac{\Psi_n(x,t)}{\psi_n(x)}\right] + 2\pi ict = 0$. Expansion of

terms leads us to the following: $(\lambda_{n+1})\ln\left[\dfrac{\Psi_n(x,t)}{\psi_n(x)}\right] + (\lambda_{n+2})\ln\left[\dfrac{\Psi_n(x,t)}{\psi_n(x)}\right] + 2\pi ict = 0$. Since

$\lambda_{n+1} = \Phi\lambda_{n+2}$ we may re-write the above equation as:

$(\Phi\lambda_{n+2})\ln\left[\dfrac{\Psi_n(x,t)}{\psi_n(x)}\right] + (\lambda_{n+2})\ln\left[\dfrac{\Psi_n(x,t)}{\psi_n(x)}\right] + 2\pi ict = 0$. Factoring out

$\lambda_{n+2}\ln\left[\dfrac{\Psi_n(x,t)}{\psi_n(x)}\right]$ we obtain $\lambda_{n+2}\ln\left[\dfrac{\Psi_n(x,t)}{\psi_n(x)}\right](\Phi+1) + 2\pi ict = 0$ or since

$(\Phi+1) = \Phi^2$ we obtain $(\Phi^2\lambda_{n+2})\ln\left[\dfrac{\Psi_n(x,t)}{\psi_n(x)}\right] = -2\pi ict$. Dividing by $\Phi^2(\lambda_{n+2})$ we get:

(28) $\ln\left[\left(\dfrac{\Psi_n(x,t)}{\psi_n(x)}\right)\right] = \dfrac{-2\pi ict}{\Phi^2\lambda_{n+2}}$. In exponential form this expression becomes:

(29) $\dfrac{\Psi_n(x,t)}{\psi_n(x)} = e^{-2\pi ict/\Phi^2\lambda_{n+2}}$ or, $\Psi_n(x,t) = \psi_n(x)e^{-2\pi ict/\Phi^2\lambda_{n+2}}$.

We have now related the three most important and ubiquitous irrational numbers e, π and Φ as well as the speed of light ***c***, time and the recursive wavelengths to the state functions of the time-independent

Schrödinger Wave Equation. Phi is the most irrational number known since it approaches itself via the famous Fibonacci Series slower than any other irrational number. The Fibonacci series is as follows: 0,1,1,2,3,5,8,13,21,34,55,89… Each number in the series is equal to the sum of the two numbers preceding it. If you take any number of the series (preferably as far out as possible) and divide it by the previous number of the series, you will obtain a value that is very close to Phi. Obviously, the further out you take the series, the closer it will get to Phi. Since Phi is an irrational number, it contains an infinite number of digits and therefore you would need to take the Fibonacci Series out to an infinite number of terms (i.e., $F^{(n)}(\infty)$) in order to obtain the "exact" value of Phi.

5. Eigenvalue Solutions to the Schrödiger wavefunctions based on Phi using the Hamiltonian operator.

The Hamiltonian operator for a wavefunction for a one particle system in a Cartesian coordinate four-dimensional space-time is defined as follows [8]:

(30) $$\hat{H} = -\sum_{i=1}^{n} \frac{\hbar^2}{2m} \nabla_i^2 + V\left(x_1, \ldots, z_n\right).$$

Using the above Hamiltonian (30) to operate on the wavefunction we obtain:

(31) $$\left[-\sum_{i=1}^{n} \frac{\hbar^2}{2m} \nabla_i^2 + V\left(x_1, \ldots, z_n\right) \right] \Psi_n\left(x_i, y_i, z_i, t\right) = E\psi_n$$

The solution to equation (31) using Mathematica 5.2 is as follows:

(32) $$\left\{ V(x_i, y_i, z_i) - \sum_{i=1}^{n} \left\{ \frac{\hbar^2}{2m} \right\} \left[\nabla_i^2 \right] \right\} \left[\left(e^{\frac{-2\pi i c t}{\Phi^2 \lambda_{n+2}}} \psi_n\left(x_i, y_i, z_i\right) \right) \right]$$

The solution to the ***Phi-Based Time-Independent Schrödinger Wave Equation*** in polar coordinates using Mathematica 5.2 is shown below in equation (33):

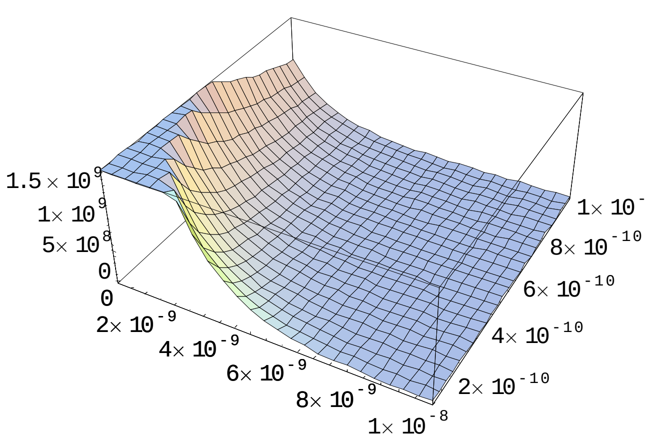
(33)

$$-V\left(r, \theta, \varphi\right) \psi_n\left(r, \theta, \varphi\right) - \frac{\hbar^2 e^{\frac{-2\pi i c t}{\Phi^2 \lambda_{n+2}}}}{2mr^2} \left[r^2 \frac{\partial^2 \psi_n}{\partial r^2} + 2r \frac{\partial \psi_n}{\partial r} + \csc^2 \theta \frac{\partial^2 \psi_n}{\partial \varphi^2} + \cot \theta \frac{\partial \psi_n}{\partial \theta} + \frac{\partial^2 \psi_n}{\partial \theta^2} \right]$$

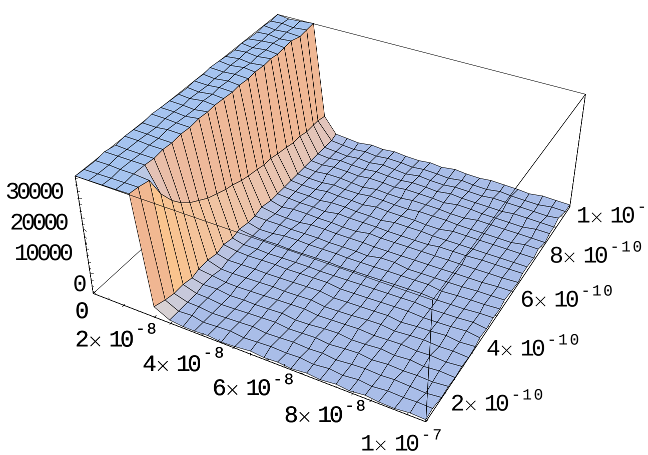
6. Three-dimensional plots of the pre-exponential factors for the Phi-Recursive wavefunctions.

Below are shown the 3D-plots of the pre-exponential factors for the Phi-Recursive wavefunctions generated using Mathematica 5.2:

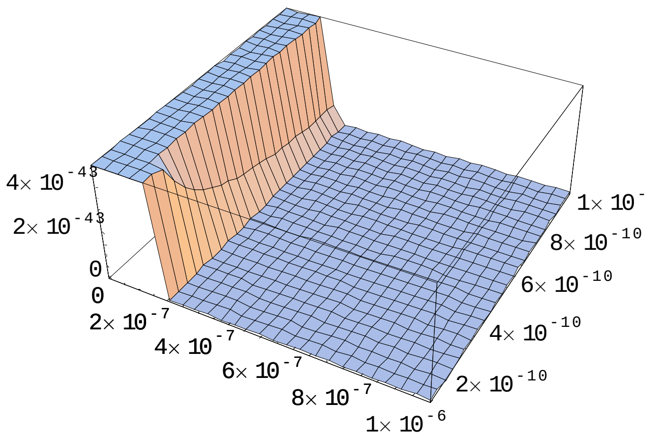
```
Plot3D[2.71828^{(-2.39996*299792458)*t}/(λn+2), {t, 0, 10^-8}, {λn+2, 10^-10, 10^-9}]
```




```
Plot3D[2.71828^{(-2.39996*299792458)*t}/(λn+2), {t, 0, 10^-7}, {λn+2, 10^-10, 10^-9}]
```



```
Plot3D[2.71828^{(-2.39996*299792458)*t}/(λ_{n+2}), {t, 0, 10^-6}, {λ_{n+2}, 10^-10, 10^-9}]
```

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Plot3D[2.71828^{(-2.39996*299792458)*t}/(λ_{n+2}), {t, 0, 10^-5}, {λ_{n+2}, 10^-10, 10^-9}]
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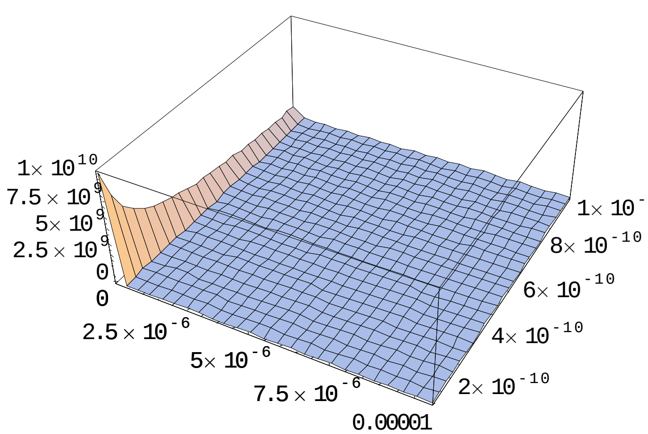
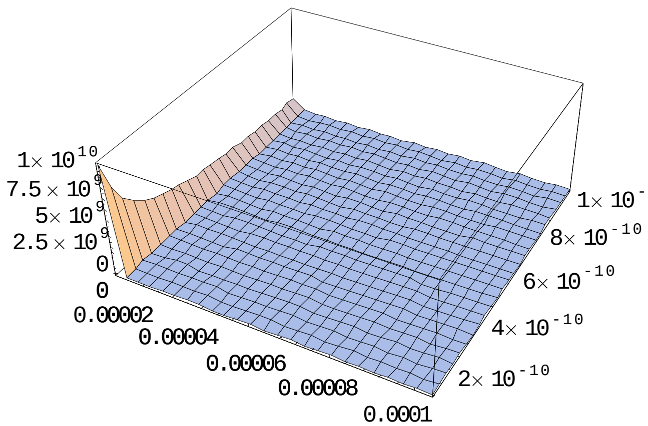
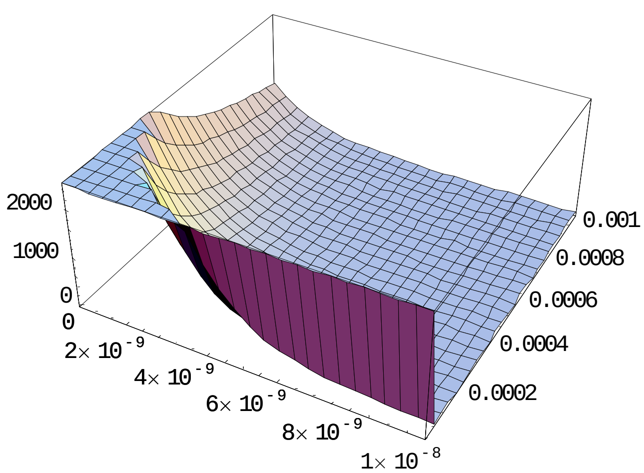


Plot3D[**2.71828^**{(**-2.39996** * **299792458**) * **t**} / (λ_{n+2}), {**t, 0, 10^-4**}, {λ_{n+2}, **10^-10, 10^-9**}]



Plot3D[**2.71828^**{(**-2.39996** * **299792458**) * **t**} / (λ_{n+2}), {**t, 0, 10^-8**}, {λ_{n+2}, **10^-8, 10^-3**}]



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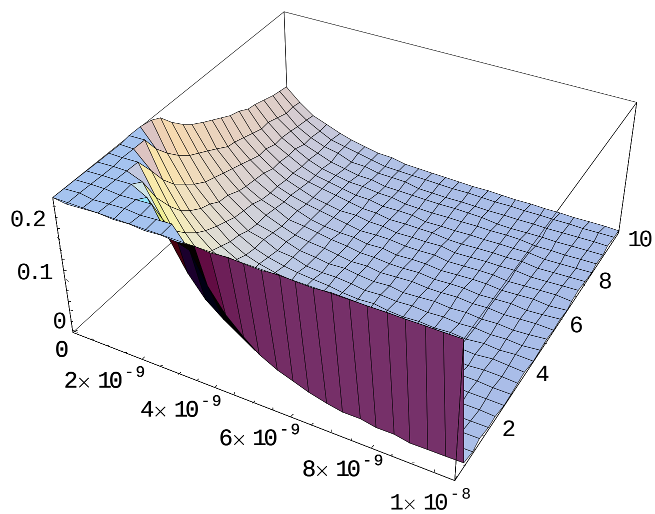
Plot3D[2.71828^{(-2.39996*299792458)*t}/(λn+2), {t, 0, 10^-8}, {λn+2, 10^-8, 10}]
```

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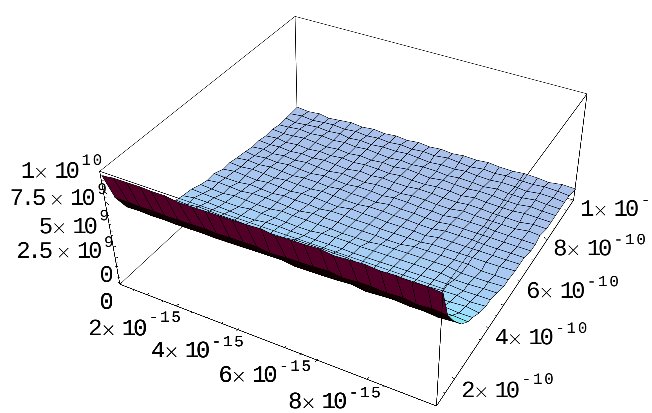
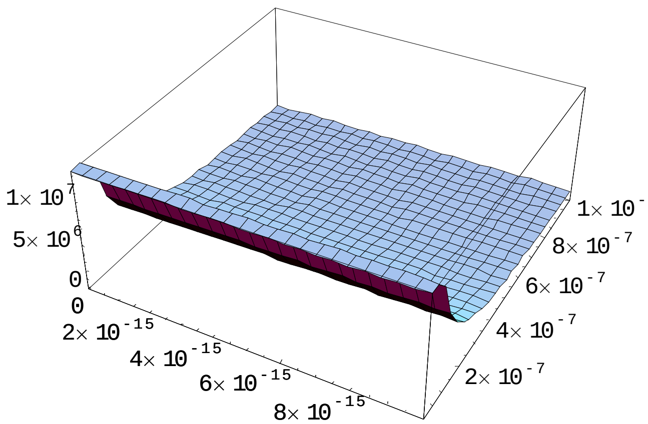
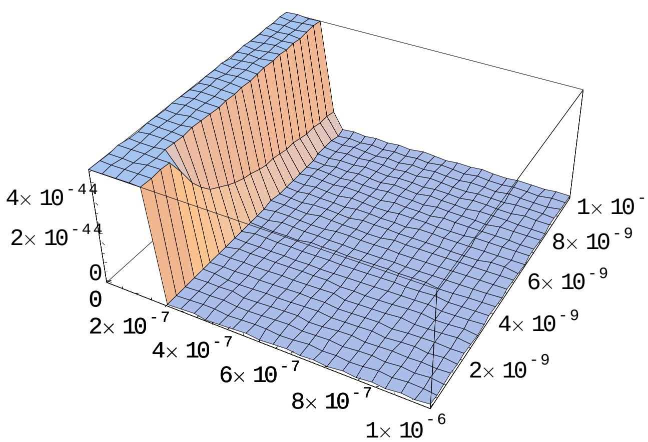


Plot3D[2.71828^{(-2.39996 * 299792458) * t} / (λ_{n+2}), {t, 0, 10^-14}, {λ_{n+2}, 10^-10, 10^-6}]



Plot3D[2.71828^{(-2.39996 * 299792458) * t} / (λ_{n+2}), {t, 0, 10^-6}, {λ_{n+2}, 10^-10, 10^-8}]



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Plot3D[2.71828^{(-2.39996*299792458)*t}/(λn+2), {t, 0, 10^-8}, {λn+2, 1000, 10000}]
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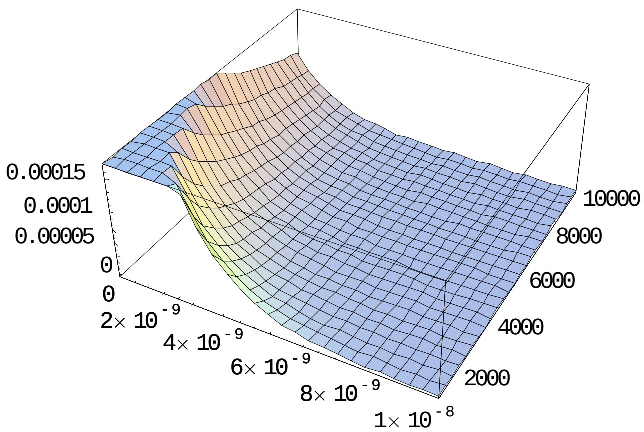

7. Conclusions:

In conclusion, it may be surmised that the state function Ψ(x, y, z, t) of the ***Time-independent Schrödinger wave equation*** is directly proportional to a pre-exponential factor containing the four basic constants (b4C), namely 2, e, π and Φ and the time-independent wavefunction ψ(x, y, z). This conclusion arises as a direct result of the incorporation of the heterodyning set of wavelengths into the actual classical Schrödinger wave equation. Also, it appears that by so doing, this paper has demonstrated that individual wavelengths and/or frequencies are actually a summation of an infinite number of wavelengths or frequencies. This concept, brought forth in this paper tends to support the so-called "Many Worlds Interpretation" (MWI) of quantum theory as opposed to the Copenhagen Interpretation whereby the "collapse" of the quantum mechanical wavefunction Ψ occurs as a result of the mere "observation" of a subatomic particle. Additionally, perhaps the most important concept resulting from this paper is that the ubiquitous irrational number Phi (Φ) is both an integral and essential constant in the quantum mechanical realm of reality.

8. Acknowledgements

The author greatly acknowledges Frank van den Bovenkamp for his derivation of the Phi-Recursive heterodyning set. I received a great deal of inspiration when I noticed van den Bovenkamp's derivation of the Phi-Recursive heterodyning set. Without this crucial piece of information, this paper would not have been possible. I therefore decided to use Frank's derivation and successfully incorporate it into the Schrödinger wave function. Please see Frank's "Why Phi" paper at http://www.heartcoherence.com/whyphi2007.pdf.

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